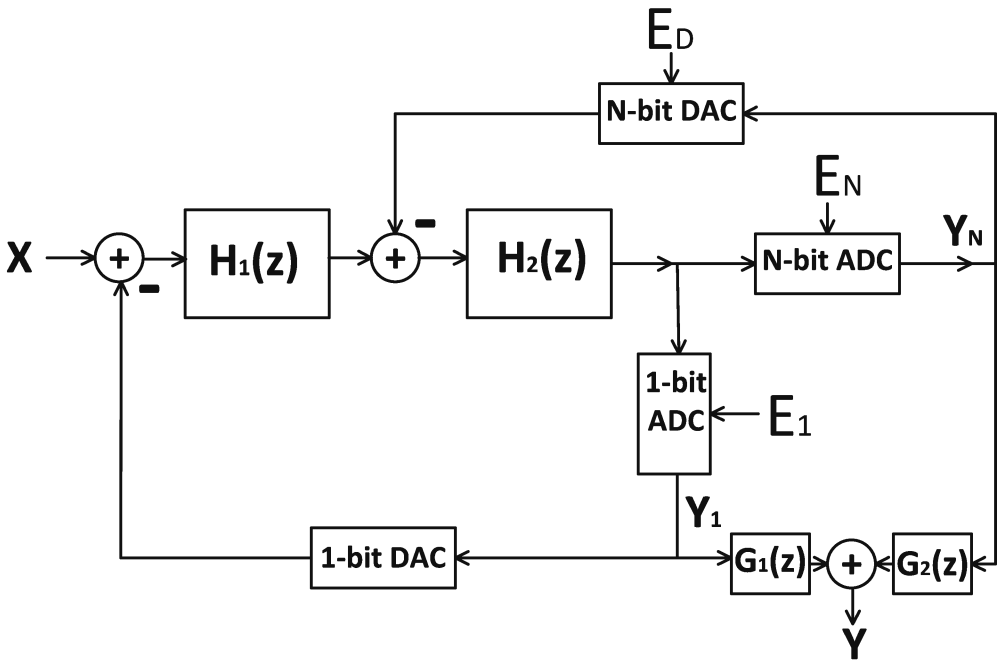
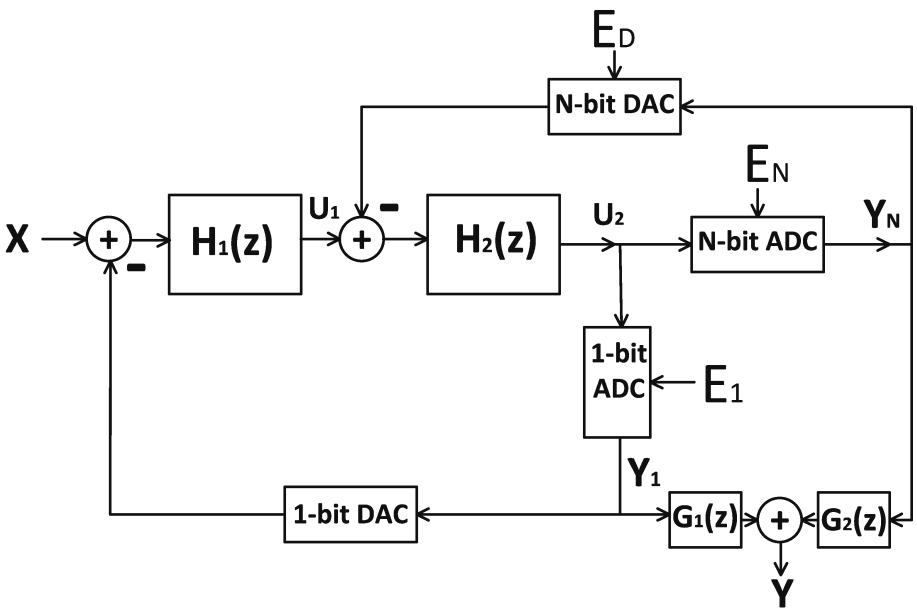
3(a) Analyze the delta-sigma loop shown. Find $\mathbf{Y_1(z)}$ and $\mathbf{Y_N(z)}$ in terms of the input **X(z)**, the quantization errors $\mathbf{E_1(z)}$ and $\mathbf{E_N(z)}$, and the DAC error $\mathbf{E_D(z)}$.

3(b) Find the transfer functions $\mathbf{G_1(z)}$ and $\mathbf{G_N(z)}$ such that **Y(z)** does not contain $\mathbf{E_1(z)}$, and **X(z)** is not distorted in the global output **Y(z)**.





$U_1=H_1(X-Y_1)$

$U_2=H_2(U_1-Y_N-E_D)=H_2(H_1X-H_1Y_1-Y_N-E_D)$

$Y_1=U_2+E_1=H_1H_2X-H_1H_2Y_1-H_2Y_N-H_2E_D+E_1$

(*)$Y_N=U_2+E_N=Y_1-E_1+E_N$

(**)$Y_1(1+H_1H_2+H_2)=H_1H_2X+(H_2+1)E_1-H_2E_N-H_2E_D$

From (*) we can derive:: (***) $Y_N[1+H_1H_2+H_2]=Y_1[1+H_1H_2+H_2]-E_1[1+H_1H_2+H_2\ ]+E_N[1+H_1H_2+H_2]$

By subsituting (**) in (***) we can derive: (****) $Y_N(1+H_1H_2+H_2)=H_1H_2X-E_1H_1H_2+E_N(1+H_1H_2)-H_2E_D$

From (**) and (****): To cancel E_1, Multiply Y_1 by G_1, Y_N by G_2

$G_1(1+H_2) = G_NH_1H_2$

$G_1 =FH_1H_2$, $G_N = F(H_2+1)$

To have X undistorted, The STF (transfer function of X) in $G_1Y_1+G_NY_N$ must be in form of z^{-k}

$\frac{(G_1+G_N)H_1H_2}{(1+H_1H_2+H_2)}$=**$FH_1H_2$**, so $\mathbf{F = \frac{z^{-k}}{H_1H_2}}$

$$\mathbf{G_N = \frac{z^{-k}(1+H_2)}{H_1H_2}}$$

$$\mathbf{G_1 = z^{-k}}$$